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Employment of continued fractions to discover solutions of the Diophantine equation ${x_n}^2 - (p^2 + Mq)\,{y_n}^2 = (Mq)^{2n}$

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Abstract - *In this study, the solutions of the Diophantine Equation* ${x_n}^2 - (p^2 + Mq)\,{y_n}^2 = (Mq)^{2n}$, $M \in Z - \{0\}$ *where p and q are twin primes, cousin primes, safe primes in terms of generalised Pell and generalised Lucas sequences by employing continued fractions are studied. Correspondingly, all the derived solutions to these categories of equations are confirmed by Python programs.*

Keywords: Generalised Pell and Pell Lucas sequence, twin primes, cousin primes, safe primes, continued fractions.

1. Introduction

Diophantine equations are those that have two or more unknowns and have an integer solution. Any Diophantine equation of the form $x^2 - Dy^2 = N$, where D is a non-square positive integer, is referred to as a Pell equation in honor of the English Mathematician John Pell. The solution of the Diophantine equation using identities of connecting sequence of numbers was studied in [7, 8]. Solutions were described in [9] using metallic ratios and continuing fractions. Several authors explored the solutions of Diophantine equations by employing various techniques [1–6, 10].

In this communication, the solutions of the Diophantine Equation ${x_n}^2 - (p^2 + Mq)\,{y_n}^2 = (Mq)^{2n}$, $M \in Z - \{0\}$ where p and q are twin primes, cousin primes, safe primes in terms of generalised Pell and generalised Lucas sequences with the help of continued fractions are deliberated. Also, all the resulting solutions to these groups of equations are examined by Python programs.

2. Identities of generalised Pell and Pell Lucas numbers

The generalized Pell and Pell–Lucas sequences, respectively, for two non-zero integers k and h performing $k^2 + h > 0$ are described by $\phi_{n+2}(k,h) = 2k\phi_{n+1}(k,h) + h\phi_n(k,h), n \geq 0$ with $\phi_0(k,h) = 0$, $\phi_1(k,h) = 2k$, $\psi_{n+2}(k,h) = 2k\psi_{n+1}(k,h) + h\psi_n(k,h), n \geq 0$ with $\psi_0(k,h) = 2, \psi_1(k,h) = 2k$.

The formulae for these sequences according to Binet are $\phi_n(k,h) = 2k\frac{\alpha^n - \beta^n}{\alpha - \beta}$, $\psi_n(k,h) = \alpha^n + \beta^n$ where $\alpha = k + \sqrt{k^2 + h}$ and $\beta = k - \sqrt{k^2 + h}$ represent the roots of the equation $x^2 - 2kx - h = 0$ such that $\alpha + \beta = 2k, \alpha - \beta = 2\sqrt{k^2 + h}$ and $\alpha\beta = -h$.

3. Recognition of integer solutions to the nature of Pell's equation

In this section, the solutions to the Diophantine equation ${x_n}^2 - (p^2 + Mq)\,{y_n}^2 = (Mq)^{2n}, M \in Z - \{0\}$ where p and q are cousin primes, twin primes, safe primes in terms of generalised Pell and Pell Lucas numbers are recognized.

Theorem 3.1

If $p, q = p + 2$ are twin primes and $D = p^2 + Mq = p^2 + M(p + 2)$ such that D is not a perfect square, then

1. $\sqrt{D} = \left[p; \overline{\frac{2p}{M(p+2)}, 2p}\right]$
2. Fundamental solution of the equation ${x_n}^2 - D{y_n}^2 = (M^2(p + 2)^2)^n$ is $(x_1, y_1) = (2p^2 + M(p + 2), 2p)$
3. All non-zero integer solutions to the equation ${x_n}^2 - D{y_n}^2 = (M^2(p + 2)^2)^n, n \geq 1$ are given by

$$x_n = \frac{1}{2}\psi_n(2p^2 + M(p + 2), -M^2(p + 2)^2)$$
$$y_n = \frac{p}{2p^2 + M(p + 2)}\phi_n(2p^2 + M(p + 2), -M^2(p + 2)^2)$$

Proof

1. $\sqrt{D} = p + \sqrt{p^2 + M(p+2)} - p$
   $= p + \cfrac{1}{\cfrac{2p}{M(p+2)} + \cfrac{\sqrt{p^2+M(p+2)}-p}{M(p+2)}}$
   $= p + \cfrac{1}{\cfrac{2p}{M(p+2)} + \cfrac{1}{2p+\sqrt{p^2+M(p+2)}-p}}$
   $= \left[p:\overline{\frac{2p}{M(p+2)}, 2p}\right]$
2. The length of period is even with a duration of 2. Then, the relation $\frac{p_1}{q_1} = p + \frac{1}{\frac{2p}{M(p+2)}} = \frac{2p^2+M(p+2)}{2p}$ gives the smallest solution to the desired equation.
3. Thus, $(x_1, y_1) = (2p^2 + Mq, 2p)$ is the basic solution to the equation $x_n{}^2 - Dy_n{}^2 = (M^2(p+2)^2)^n$.
4. Theoretically, the entire integer solutions to the equations $x_n{}^2 - Dy_n{}^2 = (M^2(p+2)^2)^n$ are expressed as follows
   $x_n + y_n\sqrt{D} = \left(2p^2 + M(p+2) + 2p\sqrt{D}\right)^n$
   $x_n - y_n\sqrt{D} = \left(2p^2 + M(p+2) - 2p\sqrt{D}\right)^n$
   The selections $\alpha = 2p^2 + M(p+2) + 2p\sqrt{D}$ and $\beta = 2p^2 + M(p+2) - 2p\sqrt{D}$ provides that
   $x_n = \frac{\alpha^n + \beta^n}{2}$
   $= \frac{1}{2}\psi_n(2p^2 + M(p+2), -M^2(p+2)^2)$
   $y_n = \frac{\alpha^n - \beta^n}{2\sqrt{D}}$
   $= \frac{p}{2p^2+Mq}\phi_n(2p^2 + M(p+2), -M^2(p+2)^2)$

The succeeding table 3.1 provides that arithmetical illustrations of theorem 3.1.

Table 3.1

p	M	n	ϕ_n	ψ_n	x_n	y_n	$x_n{}^2 - Dy_n{}^2$	$(M^2(p+2)^2)^n$
3	-2	1	16	16	8	6	100	100
		2	256	56	28	96	10000	10000
		3	2496	-704	-352	936	1000000	1000000
	-1	1	26	26	13	6	25	25
		2	676	626	313	156	625	625
		3	16926	15626	7813	3906	15625	15625
	1	1	46	46	23	6	25	25
		2	2116	2066	1033	276	625	625
		3	96186	93886	46943	12546	15625	15625
	2	1	56	56	28	6	100	100
		2	3136	2936	1468	336	10000	10000
		3	170016	158816	79408	18216	1000000	1000000
7	-2	1	160	160	80	14	324	324
		2	25600	24952	12476	2240	104976	104976
		3	4044160	3940480	1970240	353864	34012224	34012224
	-1	1	178	178	89	14	81	81
		2	31684	31522	15761	2492	6561	6561
		3	5625334	5596498	2798249	442442	531441	531441
	1	1	214	214	107	14	81	81
		2	45796	45634	22817	2996	6561	6561
		3	9783010	9748342	4874171	640010	531441	531441
	2	1	232	232	116	14	324	324
		2	53824	53176	26588	3248	104976	104976
		3	12412000	12261664	6130832	749000	34012224	34012224

The following Python program 1 can help to verify all possible solutions to the equation ${x_n}^2 - (p^2 + Mq)\, {y_n}^2 = (Mq)^{2n}$**.**

Python program 1

```
from decimal import Decimal, getcontext
# Set the precision for Decimal calculations
getcontext().prec = 40 # You can adjust the precision as needed
def lucas(n, k, t):
  if n < 0:
    return "Incorrect input"
  elif n == 0:
    return 2
  elif n == 1:
    return 2 * k
  else:
    a, b = 2, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def pell(n, k, t):
  if n < 0:
    return "Incorrect input"
  elif n == 0:
    return 0
  elif n == 1:
    return 2 * k
  else:
    a, b = 0, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def calculate_values(M, p, n):
  D = (p * p) + M * (p + 2)
  k = (2 * p * p + M * (p + 2))
  t = -(M * (p + 2)) ** 2
  x_n = lucas(n, k, t) / 2
  x2 = x_n ** 2
  y_n = p * pell(n, k, t) / k
  y2 = y_n ** 2
  D_y2 = D * y2
  h = x2 - D_y2
  return D, x_n, x2, y_n, y2, D_y2, h
# Get user inputs
M = int(input('Enter The Value Of M: '))
p = int(input('Enter The Value Of p: '))
n = int(input('Enter The Value Of n: '))
D, x_n, x2, y_n, y2, D_y2, h = calculate_values(M, p, n)
# Print the results with proper formatting using Decimal
print(f'D = {D}')
print(f'x(n) = {x_n}')
print(f'x(n)^2 = {x2}')
print(f'y(n) = {y_n}')
print(f'y(n)^2 = {y2}')
print(f'D * y(n)^2 = {D_y2}')
print(f'h = {h}')
if(h == (M * (p + 2)) ** (2 * n)):
  print("x and y are the solutions of the given equations")
```

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```
else:
  print("x and y are not the solutions of the given equations")
```

Output

$Enter\ The\ Value\ Of\ M: -2$
$Enter\ The\ Value\ Of\ p: 3$
$Enter\ The\ Value\ Of\ n: 3$
$D = -1$
$x(n) = -352.0$
$x(n)^\wedge 2 = 123904.0$
$y(n) = 936.0$
$y(n)^\wedge 2 = 876096.0$
$D * y(n)^\wedge 2 = -876096.0$
$h = 1000000.0$
$x\ and\ y\ are\ the\ solutions\ of\ the\ given\ equations$
$Enter\ The\ Value\ Of\ M: -2$
$Enter\ The\ Value\ Of\ p: 7$
$Enter\ The\ Value\ Of\ n: 3$
$D = 31$
$x(n) = 1970240.0$
$x(n)^\wedge 2 = 3881845657600.0$
$y(n) = 353864.0$
$y(n)^\wedge 2 = 125219730496.0$
$D * y(n)^\wedge 2 = 3881811645376.0$
$h = 34012224.0$
$x\ and\ y\ are\ the\ solutions\ of\ the\ given\ equations$
$Enter\ The\ Value\ Of\ M: 5$
$Enter\ The\ Value\ Of\ p: 11$
$Enter\ The\ Value\ Of\ n: 2$
$D = 186$
$x(n) = 184273.0$
$x(n)^\wedge 2 = 33956538529.0$
$y(n) = 13508.0$
$y(n)^\wedge 2 = 182466064.0$
$D * y(n)^\wedge 2 = 33938687904.0$
$h = 17850625.0$
$x\ and\ y\ are\ the\ solutions\ of\ the\ given\ equations$

Theorem 3.2

If $p, q = p + 4$ are cousin primes such that $D = p^2 + Mq$ is not a perfect square, then

1. $\sqrt{D} = \left[p; \overline{\frac{2p}{M(p+4)}, 2p}\right]$
2. Basic solution of the equation ${x_n}^2 - D{y_n}^2 = (M^2(p+4)^2)^n$ is $(x_1, y_1) = (2p^2 + M(p+4), 2p)$
3. All conceivable integer solutions (x_n, y_n) for a natural number $n \geq 1$ to the equation ${x_n}^2 - D{y_n}^2 = (M^2(p+4)^2)^n$ are specified by

$$x_n = \frac{1}{2}\psi_n(2p^2 + M(p+4), -(M^2(p+4)^2)^n)$$
$$y_n = \frac{p}{2p^2 + M(p+4)}\phi_n(2p^2 + M(p+4), -(M^2(p+4)^2)^n$$

Proof

The proof of this theorem is similar to Theorem 3.1.

Numerical solutions sustaining the equation in theorem 3.2 are enlisted in table 3.2.

Table 3.2

p	M	n	ϕ_n	ψ_n	x_n	y_n	${x_n}^2 - D{y_n}^2$	$(M^2(p+4)^2)^n$
3	-2	1	8	8	4	6	196	196
		2	64	-328	-164	48	38416	38416
		3	-1056	-4192	-2096	-792	7529536	7529536

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	-1	1	22	22	11	6	49	49
		2	484	386	193	132	2401	2401
		3	9570	7414	3707	2610	117649	117649
	1	1	50	50	25	6	49	49
		2	2500	2402	1201	300	2401	2401
		3	122550	117650	58825	14706	117649	117649
	2	1	64	64	32	6	196	196
		2	4096	3704	1852	384	38416	38416
		3	249600	224512	112256	23400	7529536	7529536
5	-2	1	64	64	32	10	324	324
		2	4096	3448	1724	640	104976	104976
		3	241408	199936	99968	37720	34012224	34012224
	-1	1	82	82	41	10	81	81
		2	6724	6562	3281	820	6561	6561
		3	544726	531442	265721	66430	531441	531441
	1	1	118	118	59	10	81	81
		2	13924	13762	6881	1180	6561	6561
		3	1633474	1614358	807179	138430	531441	531441
	2	1	136	136	68	10	324	324
		2	18496	17848	8924	1360	104976	104976
		3	2471392	2383264	1191632	181720	34012224	34012224

The complete solutions to the equation $x_n^2 - (p^2 + M(p+4))y_n^2 = (M^2(p+4)^2)^n$ can be confirmed by Python program 2.

Python program 2

```
from decimal import Decimal, getcontext
# Set the precision for Decimal calculations
getcontext().prec = 60 # You can adjust the precision as needed
def lucas(n, k, t):
  if n < 0:
    return "Incorrect input"
  elif n == 0:
    return 2
  elif n == 1:
    return 2 * k
  else:
    a, b = 2, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def pell(n, k, t):
  if n < 0:
    return "Incorrect input"
  elif n == 0:
    return 0
  elif n == 1:
     return 2 * k
  else:
    a, b = 0, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def calculate_values(M, p, n):
  D = (p * p) + M * (p + 4)
  k = (2 * p * p + M * (p + 4))
  t = -(M * (p + 4)) ** 2
  x_n = lucas(n, k, t) / 2
```

```
    x2 = x_n ** 2
    y_n = p * pell(n,k,t) / k
    y2 = y_n ** 2
    D_y2 = D * y2
    h = x2 − D_y2
    return D,x_n,x2,y_n,y2,D_y2,h
  # Get user inputs
  M = int(input('Enter The Value Of M: '))
  p = int(input('Enter The Value Of p: '))
  n = int(input('Enter The Value Of n: '))
  D,x_n,x2,y_n,y2,D_y2,h = calculate_values(M,p,n)
  # Print the results with proper formatting using Decimal
  print(f'D = {D}')
  print(f'x(n) = {x_n}')
  print(f'x(n)^2 = {x2}')
  print(f'y(n) = {y_n}')
  print(f'y(n)^2 = {y2}')
  print(f'D * y(n)^2 = {D_y2}')
  print(f'h = {h}')
  if(h == (M * (p + 4)) ** (2 * n)):
    print("x and y are the solutions of the given equations")
  else:
    print("x and y are not the solutions of the given equations")
```

Output

```
Enter The Value Of M: 6
Enter The Value Of p: 3
Enter The Value Of n: 3
D = 51
x(n) = 546480.0
x(n)^2 = 298640390400.0
y(n) = 75816.0
y(n)^2 = 5748065856.0
D * y(n)^2 = 293151358656.0
h = 5489031744.0
```

x and y are the solutions of the given equations

```
Enter The Value Of M: 8
Enter The Value Of p: 5
Enter The Value Of n: 2
D = 97
x(n) = 24584.0
x(n)^2 = 604373056.0
y(n) = 2440.0
y(n)^2 = 5953600.0
D * y(n)^2 = 577499200.0
h = 26873856.0
x and y are the solutions of the given equations
Enter The Value Of M: 9
Enter The Value Of p: 13
Enter The Value Of n: 2
D = 322
x(n) = 458753.0
x(n)^2 = 210454315009.0
y(n) = 25532.0
y(n)^2 = 651883024.0
D * y(n)^2 = 209906333728.0
h = 547981281.0
```

x and y are the solutions of the given equation

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Theorem 3.3
If $p, q = 2p + 1$ are safe primes and $D = p^2 + Mq$ such that D is not a perfect square, then

1. $\sqrt{D} = \left[p : \overline{\frac{2p}{M(2p+1)}, 2p}\right]$
2. The least solution of the equation $x_n{}^2 - Dy_n{}^2 = (M^2(2p+1)\ ^2)^n$ is $(x_1, y_1) = (2p^2 + M(2p+1), 2p)$
3. The patterns of integer solutions (x_n, y_n) for a natural number $n \geq 1$ to the equation $x_n{}^2 - Dy_n{}^2 = (M^2(2p+1)\ ^2)^n$ are prearranged by
$$x_n = \frac{1}{2}\psi_n(2p^2 + M(2p+1), -(M^2(2p+1)\ ^2)^n)$$
$$y_n = \frac{p}{2p^2+M(2p+1)} F_n(2p^2 + M(2p+1), -(M^2(2p+1)^2)^n)$$

Proof
The proof is identical to theorem 3.1.
Numerical samples of solutions to the equation in theorem 3.3 are offered in table 3.3.

Table 3.3

p	M	n	ϕ_n	ψ_n	x_n	y_n	$x_n{}^2 - Dy_n{}^2$	$M^2(2p+1)^2$
3	-2	1	8	8	4	6	196	196
		2	64	-328	-164	48	38416	38416
		3	-1056	-4192	-2096	-792	7529536	7529536
	-1	1	22	22	11	6	49	49
		2	484	386	193	132	2401	2401
		3	9570	7414	3707	2610	117649	117649
	1	1	50	50	25	6	49	49
		2	2500	2402	1201	300	2401	2401
		3	122550	117650	58825	14706	117649	117649
	2	1	64	64	32	6	196	196
		2	4096	3704	1852	384	38416	38416
		3	249600	224512	112256	23400	7529536	7529536
5	-2	1	56	56	28	10	484	484
		2	3136	2168	1084	560	234256	234256
		3	148512	94304	47152	26520	113379904	113379904
	-1	1	78	78	39	10	121	121
		2	6084	5842	2921	780	14641	14641
		3	465114	446238	223119	59630	1771561	1771561
	1	1	122	122	61	10	121	121
		2	14884	14642	7321	1220	14641	14641
		3	1801086	1771562	885781	147630	1771561	1771561
	2	1	144	144	72	10	484	484
		2	20736	19768	9884	1440	234256	234256
		3	2916288	2776896	1388448	202520	113379904	113379904

All plausible solutions to the equation $x_n{}^2 - (p^2 + M(2p+1))y_n{}^2 = (M^2(2p+1)\ ^2)^n$ are attested by python program 3.
Python program 3

```
from decimal import Decimal, getcontext
    # Set the precision for Decimal calculations
    getcontext().prec = 60 # You can adjust the precision as needed
    def lucas(n, k, t):
      if n < 0:
        return "Incorrect input"
      elif n == 0:
        return 2
      elif n == 1:
        return 2 * k
      else:
```

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```
    a, b = 2, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def pell(n, k, t):
  if n < 0:
    return "Incorrect input"
  elif n == 0:
    return 0
  elif n == 1:
    return 2 * k
  else:
    a, b = 0, 2 * k
    for _ in range(2, n + 1):
      a, b = b, 2 * k * b + t * a
    return b
def calculate_values(M, p, n):
  D = (p * p) + M * (2 * p + 1)
  k = (2 * p * p + M * (2 * p + 1))
  t = -(M * (2 * p + 1)) ** 2
  x_n = lucas(n, k, t) / 2
  x2 = x_n ** 2
  y_n = p * pell(n, k, t) / k
  y2 = y_n ** 2
  D_y2 = D * y2
  h = x2 - D_y2
  return D, x_n, x2, y_n, y2, D_y2, h
# Get user inputs
M = int(input('Enter The Value Of M: '))
p = int(input('Enter The Value Of p: '))
n = int(input('Enter The Value Of n: '))
D, x_n, x2, y_n, y2, D_y2, h = calculate_values(M, p, n)
# Print the results with proper formatting using Decimal
print(f'D = {D}')
print(f'x(n) = {x_n}')
print(f'x(n)^2 = {x2}')
print(f'y(n) = {y_n}')
print(f'y(n)^2 = {y2}')
print(f'D * y(n)^2 = {D_y2}')
print(f'h = {h}')
if(h == (M * (2 * p + 1)) ** (2 * n)):
  print("x and y are the solutions of the given equation")
else:
  print("x and y are not the solutions of the given equation")
```

Output

```
Enter The Value Of M: 11
Enter The Value Of p: 3
Enter The Value Of n: 3
D = 86
x(n) = 1739735.0
x(n)^2 = 3026677870225.0
y(n) = 181026.0
y(n)^2 = 32770412676.0
D * y(n)^2 = 2818255490136.0
h = 208422380089.0
x and y are the solutions of the given equation
Enter The Value Of M: 5
Enter The Value Of p: 7
```

$Enter\ The\ Value\ Of\ n: 3$
$D = 124$
$x(n) = 17791493.0$
$x(n)^\wedge 2 = 316537223169049.0$
$y(n) = 1597274.0$
$y(n)^\wedge 2 = 2551284231076.0$
$D * y(n)^\wedge 2 = 316359244653424.0$
$h = 177978515625.0$
$x\ and\ y\ are\ the\ solutions\ of\ the\ given\ equation$
$Enter\ The\ Value\ Of\ M: 8$
$Enter\ The\ Value\ Of\ p: 11$
$Enter\ The\ Value\ Of\ n: 2$
$D = 305$
$x(n) = 329096.0$
$x(n)^\wedge 2 = 108304177216.0$
$y(n) = 18744.0$
$y(n)^\wedge 2 = 351337536.0$
$D * y(n)^\wedge 2 = 107157948480.0$
$h = 1146228736.0$
$x\ and\ y\ are\ the\ solutions\ of\ the\ given\ equation$

5. Conclusion

The Diophantine equation ${x_n}^2 - (p^2 + Mq)\,{y_n}^2 = (M^2q^2)^n, M \in Z - \{0\}$ in which p and q are twin primes, cousin primes, safe primes is analysed for solutions in terms of the generalised Pell and Pell Lucas sequences. Similarly, one may look for integer solutions to related kinds of Diophantine equations in terms of Jacobsthal and Jacobsthal Lucas sequences.

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